# On amplitudes in self-dual sector of Yang-Mills theory

**A.A.Rosly and K.G.Selivanov**

ITEP,B.Cheremushkinskaya 25, Moscow,117259, Russia



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**Abstract**

Self-dual perturbiner in the Yang-Mills theory is constructed by the twistor methods both in topologically trivial and topologically nontrivial cases. Maximally helicity violating amplitudes and their instanton induced analogies are briefly discussed.

1.The tree and one-loop amplitudes in the self-dual (SD) sector of the Yang-Mills (YM) theory, also known as like-helicity multi-gluon amplitudes, have been intensively studied in the literature (see, e.g., reviews [1], [2] and refs. [3]-[8] where the amplitudes were discussed from some different points of view). From our point of view, such amplitudes provide one of the most interesting illustrations of the concept of *perturbiner* introduced in ref.[9]. The perturbiner is a generating function for (a class of) tree amplitudes in the theory. As explained in ref.[9], the perturbiner can be given intrinsic definition which is formally independent of considering Feynman diagrams. It is a solution of the field equations which uniquely corresponds to a given plane waves solution of the free (zero coupling constant) field equations. The set of plane waves included in the solution of the free field equations is essentially the set of asymptotic states in the amplitudes which the perturbiner is the generating function for. Considering the asymptotic states of only negative helicity leads to considering only SD solutions of the YM equations.

In this letter we construct the SD perturbiner for the YM theory with arbitrary gauge group. We find a first order anti-SD deformation of the SD perturbiner which allows us to describe amplitudes with any number of gluons from the SD sector and with two gluons from the anti-SD sector. We find fermionic deformations of the SD perturbiner. The possibility to have an arbitrary instanton background is also taken into account.

In the topologically trivial case the SD perturbiner reproduces the prominent expressions from refs.[10], [11] for the like-helicity amplitudes (or, rather, for the like-helicity "currents", i.e. objects with a number of on-shell same helicity gluons and one arbitrary of-shell gluon). This type of SD solutions has been discussed in refs.[5], [6], [12]. In ref.[5] and independently in ref.[6], the tree like-helicity amplitudes were related to solutions of the SD equations. In [5] it was basically shown that the SD equations reproduce the recursion relations obtained originally in ref.[11] from the Feynman diagrams; the corresponding solution of SD equations was obtained with use of the known solution of refs.[10], [11] of the recursion relations. In ref.[6] an example of SD perturbiner was obtained in the SU(2) case by a 'tHooft anzatz upon further restriction on the asymptotic states included. The consideration of ref.[12] is based on solving recursion relations analogous to refs. [11].

In the topologically nontrivial case the SD perturbiner generates instanton mediated negative helicity amplitudes, closed expressions for which, as far as we know, have been absent in the literature. A particular example of the topologically nontrivial SD perturbiner for the SU(2) case was described in ref.[9]. A topologically nontrivial perturbiner in 2D Kähler sigma model has been considered in ref.[9].

2. We adopt the spinor notations, i.e., $A_{\alpha\dot\alpha}, \alpha = 1, 2, \dot\alpha = \dot 1, \dot 2$ stands for the YM potential. The perturbiner is a complex solution of the field equations. In the spinor notations the curvature form, $F = dA + A^2$, has four indices, $F_{\alpha\dot\alpha\beta\dot\beta}$, and, being antisymmetric with respect to permutation $(\alpha\dot\alpha) \leftrightarrow (\beta\dot\beta)$, decomposes as

$$F_{\alpha\dot\alpha\beta\dot\beta} = \varepsilon_{\alpha\beta}F_{\dot\alpha\dot\beta} + \varepsilon_{\dot\alpha\dot\beta}F_{\alpha\beta} \quad (1)$$

where $\varepsilon_{\alpha\beta}, \varepsilon_{\dot\alpha\dot\beta}$ are the usual antisymmetric tensors. The first term in the r.h.s. of Eq. (1) can be identified as an SD component of $F$, the second one - as an anti-SD component of $F$. Correspondingly, the SD equation can be written as

$$F_{\alpha\beta} = 0 \tag{2}$$

and the anti-SD equation - as

$$F_{\dot\alpha\dot\beta} = 0 \tag{3}$$

The free SD (anti-SD) equation is the same as Eq. (2) (Eq. (3)) with $F = dA + A^2$ substituted by $F^{(0)} = dA^{(0)}$. A free solution consisting of $N$ plane SD-waves is as follows

$$A^{(0)N}_{\alpha\dot\alpha} = \sum_j^N A_{\alpha\dot\alpha}(j) \tag{4}$$

where the sum runs over gluons, $N$ is the number of gluons, $A_{\alpha\dot\alpha}(j) = ia_j t_j \epsilon^{+j}_{\alpha\dot\alpha} e^{ik^j_{\alpha\dot\alpha}x^{\alpha\dot\alpha}}$, $x^{\alpha\dot\alpha}$ represents the space-time coordinate, $k^j_{\alpha\dot\alpha}$ is a light-like four momentum of the $j$-th gluon, $\epsilon^{+j}_{\alpha\dot\alpha}$ is a four-vector defining a polarization of the $j$-th gluon, $t_j$ is a matrix defining colour orientation of the $j$-th gluon, $a_j$ is the symbol of annihilation/creation operator (depending on the sign of the time component of $k^j_{\alpha\dot\alpha}$) of the $j$-th gluon. As in ref.[9], we assume nilpotency of $a_j$, that is $a_j^2 = 0$. Since $N$ is arbitrary, the nilpotency can be assumed without any loss of generality. Below we use the following short-cut notations

$$\begin{aligned} \mathcal{E}^j &= a_j e^{ik^j_{\alpha\dot\alpha}x^{\alpha\dot\alpha}} \\ \hat{\mathcal{E}}^j &= t_j \mathcal{E}^j \end{aligned} \tag{5}$$

$k^j_{\alpha\dot\alpha}$, as a light-like four-vector, decomposes into a product of two spinors

$$k^j_{\alpha\dot\alpha} = æ^j_\alpha \lambda^j_{\dot\alpha} \tag{6}$$

Since we are considering SD-waves, $\epsilon^{+j}_{\alpha\dot\alpha}$ is also a light-like four-vector of the following form

$$\epsilon^{+j}_{\alpha\dot\alpha} = \frac{q^j_\alpha \lambda^j_{\dot\alpha}}{(æ^j, q^j)} \tag{7}$$

where normalization is defined with use of a convolution $(æ^j, q^j) = \varepsilon^{\gamma\delta} æ^j_\gamma q^j_\delta = {æ^j}^\delta {q^j}_\delta$.

The free anti-SD equation would give rise to a polarization $\epsilon^-_{\alpha\dot\alpha}$,

$$\epsilon^-_{\alpha\dot\alpha} = \frac{æ_\alpha \bar{q}_{\dot\alpha}}{(\lambda, \bar{q})} \tag{8}$$

The auxiliary spinors $q_\alpha$ and $\bar{q}_{\dot\alpha}$ form together a four-vector $q_{\alpha\dot\alpha} = q_\alpha \bar{q}_{\dot\alpha}$ usually called a reference momentum. The normalization was chosen so that

$$\epsilon^+ \cdot \epsilon^- = \varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\epsilon^+_{\alpha\dot\alpha}\epsilon^-_{\beta\dot\beta} = -1 \tag{9}$$

By definition (cf. ref.[9]), the perturbiner $A^{ptb}$ is a solution of the field equations, of the SD equations in the present case, which is polynomial in the nilpotent symbols $\mathcal{E}^j$. In the topologically trivial case the first order term of the polynomial $A^{ptb}$ is equal to $A^{(0)}$ from Eq. (4), while the zeroth order term is absent. In the topologically nontrivial case the zeroth order term of $A^{ptb}$ is equal to an instanton field and the first order term far from the center of the instanton is the gauge transformed $A^{(0)}$ from Eq. (4) with the same gauge transformation which defines the asymptotic of the instanton.

Below we shall assume that the pertrubiner $A^{ptb}$ obeys the Bose symmetry property which means that it is invariant under permutations of the type $(\hat{\mathcal{E}}^{j_1}, æ^{j_1}, \lambda^{j_1}, q^{j_1}) \leftrightarrow (\hat{\mathcal{E}}^{j_2}, æ^{j_2}, \lambda^{j_2}, q^{j_2})$ and the restriction property

$$A_N^{ptb}|_{(\mathcal{E}^N=0)} = A_{N-1}^{ptb} \tag{10}$$

where the subscript $N$ indicates the number of gluons included. Eq. (10), in particular, means that $A_{N-1}^{ptb}$ is independent of quantum numbers of the $N$-th gluon.

One can see that such a solution is unique up to gauge transformations (cf. ref.[9]).

Before describing the solution, we introduce some more notation. With use of the auxiliary twistor variables, $p^\alpha, \alpha = 1, 2$, which can be viewed on as a pair of complex numbers, we form objects

$$\begin{aligned} A_{\dot\alpha} &= p^\alpha A_{\alpha\dot\alpha}, \\ \bar\partial_{\dot\alpha} = p^\alpha \partial_{\alpha\dot\alpha}, \text{ where } \partial_{\alpha\dot\alpha} &= \frac{\partial}{\partial x^{\alpha\dot\alpha}}, \\ \bar\nabla_{\dot\alpha} &= \bar\partial_{\dot\alpha} + A_{\dot\alpha} \end{aligned} \tag{11}$$

The SD equation, Eq. (2), then takes the form of a zero-curvature condition (ref.[13])

$$[\bar\nabla_{\dot\alpha}, \bar\nabla_{\dot\beta}] = 0, \text{ at any } p^\alpha, \alpha = 1, 2 \tag{12}$$

which is solved as

$$A_{\dot\alpha} = g^{-1}\bar\partial_{\dot\alpha} g \tag{13}$$

where $g$ is a function of $x^{\alpha\dot\alpha}$ and $p^\alpha$ with values in the complexification of the gauge group. $g$ must depend on $p^\alpha$ in such a way that the resulting $A_{\dot\alpha}$ is a linear homogeneous function of $p^\alpha$, as in Eq. (11). Probably, it is worth to stress that $A_{\dot\alpha}$ from Eq. (13) is not necessary a pure gauge since $g$ is $p^\alpha$-dependent.

Below we explicitly construct such a matrix $g^{ptb}$ for the SD perturbiner. Notice that $g^{ptb}$ inherits the Bose symmetry and restriction properties.

3. In the topologically trivial sector the perturbiner depends on $x^{\alpha\dot\alpha}$ only via $\hat{\mathcal{E}}^j, j = 1, \ldots, N$, so that instead of an infinite-dimentional function space we deal with a finite-dimensional space of polynomials in $N$ nilpotent variables. Using the nilpotency of $\hat{\mathcal{E}}^j$ and the restriction property Eq. (10) one can see that

$$g_N^{ptb} = g_{N-1}^{ptb}(1 + \chi_N) \tag{14}$$

where $\chi_N$ is of first order in $\hat{\mathcal{E}}^N$ and polynomial in all $\hat{\mathcal{E}}^j$. Here $g^{ptb}$ is assumed to be a rational function of $p^\alpha$. The linear part of $\chi_N$ is fixed by the condition that the linear part of $A^{ptb}$ is $A^{(0)}$ from Eq. (4) and the higher order terms in $\chi_N$ are fixed by demanding regularity of $A^{ptb}$ as a function of $p^\alpha, \alpha = 1, 2$. More precisely, the above conditions fix $\chi_N$ up to a freedom which is equivalent to the gauge freedom. There is, however, a minimal choice for $\chi_N$ which happens to correspond to the Lorentz gauge for $A^{ptb}$. Upon that choice one gets

$$\chi_N = \frac{(p, q^N)}{(p, æ^N)} h_{N-1}^{-1} \frac{\hat{\mathcal{E}}^N}{(æ^N, q^N)} h_{N-1} \tag{15}$$

where we have introduced $h_{N-1} = g^{ptb}_{N-1}|_{(p=æ^N)}$. With $\chi_N$ from Eq. (15), Eq. (14) becomes a recursion relation for $g^{ptb}$. This recursion relation greatly simplifies if one considers ordered [1] highest degree monomials in $g^{ptb}$, say,

$$g^{ptb}_{N(N,\ldots,1)} = C^j(æ, q) \hat{\mathcal{E}}^N \ldots \hat{\mathcal{E}}^1 \tag{16}$$

for which the recursion relation readily leads to

$$g_{N(N,\ldots,1)} = \frac{(p, q^N)(æ^N, q^{N-1}) \ldots (æ^2, q^1)}{(p, æ^N)(æ^N, æ^{N-1}) \ldots (æ^2, æ^1)} \frac{\hat{\mathcal{E}}^N}{(æ^N, q^N)} \cdots \frac{\hat{\mathcal{E}}^1}{(æ^1, q^1)} \tag{17}$$

This is, essentially, a solution of the problem. The whole $g^{ptb}$ is restored with use of the Bose symmetry and the restriction property,

$$g^{ptb} = \sum_{d=0} \sum_{J_d} \frac{(p, q^{j_d})(æ^{j_d}, q^{j_{d-1}}) \ldots (æ^{j_2}, q^{j_1})}{(p, æ^{j_d})(æ^{j_d}, æ^{j_{d-1}}) \ldots (æ^{j_2}, æ^{j_1})} \frac{\hat{\mathcal{E}}^{j_d}}{(æ^{j_d}, q^{j_d})} \cdots \frac{\hat{\mathcal{E}}^{j_1}}{(æ^{j_1}, q^{j_1})} \tag{18}$$

where the second sum runs over all ordered subsets $J_d = \{j_1, \ldots, j_d\}$ of the set $\{1, \ldots, N-1\}$.

Substituting $g^{ptb}$ from Eq. (18) into Eq. (13) determines the perturbiner $A^{ptb}_{\dot\alpha}$. At first glance, the corresponding computation looks somewhat cumbersome, but again there is a short-cut. By construction, $A^{ptb}_{\dot\alpha}$ is a linear homogeneous function of $p^\alpha$. Finding such a function is equivalent to finding its derivative with respect to $p^\alpha$ at any value of $p^\alpha$. We put at this moment all $q$'s equal each other (it is a gauge choice for the gluons) and compute the derivative of $A^{ptb}_{\dot\alpha}$ at $p = q$. In this case $g^{ptb}|_{(p=q)} = 1$, and the perturbiner sought-for is

$$A^{ptb}_{\alpha\dot\alpha} = i \sum_{d=1} \sum_{J_d} \frac{\left(\sum_{l \in J_d} q_\alpha (æ^l, q) \lambda^l_{\dot\alpha}\right)}{(æ^{j_d}, q)(æ^{j_1}, q)} \frac{\hat{\mathcal{E}}^{j_d} \ldots \hat{\mathcal{E}}^{j_1}}{(æ^{j_d}, æ^{j_{d-1}}) \ldots (æ^{j_2}, æ^{j_1})} \tag{19}$$

4. The SD perturbiner can be used as a base point for a perturbation procedure of adding one-by-one gluons of the opposite helicity, or other particles,

[1] One can see that the decomposition of any polynomial in $\hat{\mathcal{E}}^j$'s into the ordered monomials is well defined.

say, fermions, interacting with gluons (cf. ref.[9], [6]). The explicit expression for $g^{ptb}$, Eq. (18), obtained above is very useful in this procedure.

The SD perturbiner itself describes the so-called off-shell currents - objects including an arbitrary number of on-shell SD gluons and one arbitrary off-shell gluon. When the latter gluon becomes on-shell, one gets an amplitude with all gluons but one having the same helicity and the latter one having an arbitrary helicity. Such amplitudes are known to be zero (see, e.g., the review [1]). That vanishing can be seen applying to the perturbiner Eq. (19) the reduction formula ref.[9], [6]

$$M(k'', \{a_j\}) = -i \int d^4x \, tr[(dA'') \cdot (dA^{ptb})] \tag{20}$$

where $M$ is a generating function for the amplitudes with one marked gluon of arbitrary helicity, $k''$ stands for the quantum numbers of the marked gluon, $A''$ is the corresponding solution of the free equations, $d$ stands for the external derivative, $\cdot$ indicates the scalar product defined by the space-time metric, $tr$ is the trace. The amplitudes with one marked particle are generated as coefficients in expansion of $M$ in powers of the symbols $a_j$.

To include one more gluon of the opposite helicity one needs to find a first order anti-SD deformation of the SD perturbiner, that is, to solve the linearized YM equation in the background of SD perturbiner. Its solution goes as follows.

The YM equations

$$\begin{aligned} \nabla * F &= 0 \\ \nabla F &= 0 \end{aligned} \tag{21}$$

rewrite as

$$\begin{aligned} \varepsilon^{\alpha\beta} \nabla_{\alpha\dot{\alpha}} F_{\beta\gamma} &= 0 \\ \varepsilon^{\dot{\alpha}\dot{\beta}} \nabla_{\alpha\dot{\alpha}} F_{\dot{\beta}\dot{\gamma}} &= 0 \end{aligned} \tag{22}$$

At first step one solves the variation of the first of Eqs. (22),

$$\varepsilon^{\alpha\beta} \nabla_{\alpha\dot{\alpha}} f_{\beta\gamma} = 0 \tag{23}$$

where $f_{\beta\gamma} = \delta F_{\beta\gamma}$. $f_{\beta\gamma}$ must be of the first order in $\hat{\mathcal{E}}'$ and polynomial in $\hat{\mathcal{E}}^j$, $j = 1, \dots, N$, where $\hat{\mathcal{E}}'$ corresponds to the added anti-SD gluon. As above, the first order term, $f^{(0)}_{\beta\gamma}$, in the polynomial $f_{\beta\gamma}$ must be a solution of the free anti-SD equation,

$$f^{(0)}_{\alpha\beta} = æ'_\alpha æ'_\beta \hat{\mathcal{E}}', \tag{24}$$

the corresponding four-momentum $k'_{\alpha\dot{\alpha}}$ being $k'_{\alpha\dot{\alpha}} = æ'_\alpha \lambda'_{\dot{\alpha}}$.

At second step one finds a potential $a_{\alpha\dot{\alpha}}$ such that anti-SD component of its covariant derivative is the above $f_{\alpha\beta}$

$$(\nabla a)_{\alpha\beta} = f_{\alpha\beta} \tag{25}$$

where the covariant derivative $\nabla_{\alpha\dot\alpha}$ is in the SD perturbiner background, $A^{ptb}_{\dot\alpha} = (g^{ptb})^{-1}\bar\partial_{\dot\alpha} g^{ptb}$ .

The first step is readily done,

$$f_{\alpha\beta} = (g^{ptb}|_{(p=æ')})^{-1} f^{(0)}_{\alpha\beta}\, g^{ptb}|_{(p=æ')} \tag{26}$$

As concernes the second step, one luckily need not doing it. Indeed, the reduction formula relating amplitudes with two marked gluons to the deformation $a_{\alpha\dot\alpha}$ of the perturbiner reads

$$M(k'', k', \{a_j\}) = -i \int d^4x\, tr[(dA'') \cdot (da)] \tag{27}$$

where $M$ is a generating function for such amplitudes, $k'', k'$ stand for the quantum numbers of the marked gluons, $k'$ entering $a$, the solution of the second step of the problem above. Assuming the doubly primed gluon to be anti-SD Eq. (20) rewrites as

$$M(k'', k', \{a_j\}) = -i \int d^4x\, tr[\hat{\mathcal{E}}'' æ''^{\alpha} æ''^{\beta} \partial_{\alpha\dot\alpha} a_{\beta\dot\beta} \varepsilon^{\dot\alpha\dot\beta}] \tag{28}$$

where $\hat{\mathcal{E}}''$ corresponds to the doubly primed gluon, the corresponding four momentum $k''_{\alpha\dot\alpha}$ decomposing as $k''_{\alpha\dot\alpha} = æ''_{\alpha}\lambda''_{\dot\alpha}$

Taking the reference momentum, $q$, in the perturbiner $A^{ptb}$ such that $(æ'', q) = 0$ one can substitute in the r.h.s. of Eq. (28) the derivative $\partial_{\alpha\dot\alpha}$ by the covariant derivative $\nabla_{\alpha\dot\alpha}$ in the $A^{ptb}$ background. This way, with use of the solution Eq. (26), one comes to a compact expression for the generating function of the Parke-Taylor [10] amplitudes

$$M(k'', k', \{a_j\}) = -i(æ'', æ')^2 \int d^4x\, tr[\hat{\mathcal{E}}''(g^{ptb}|_{(p=æ')})^{-1} \hat{\mathcal{E}}' g^{ptb}|_{(p=æ')}] \tag{29}$$

In verifying the equivalence of Eq. (29) to the Parke-Taylor expressions [10] we recommend to consider the terms of definite cyclic order in $\hat{\mathcal{E}}$'s..

To include into the game a couple of fermions is as easy as to include a couple of anti-SD gluons, because the fermion field equations are analogous to Eq. (23). More precisely, depending on chirality of the fermions, the field equations look as

$$\varepsilon^{\alpha\beta}\nabla_{\alpha\dot\alpha}\Psi_\beta = 0 \tag{30}$$

or as

$$\varepsilon^{\dot\alpha\dot\beta}\nabla_{\alpha\dot\alpha}\Psi_{\dot\beta} = 0 \tag{31}$$

where $\nabla_{\alpha\dot\alpha}$ is in background of the SD perturbiner. Again the solution $\Psi_\beta$ ($\Psi_{\dot\beta}$) must be of the first order in $\mathcal{E}'$ and polynomial in $\hat{\mathcal{E}}^j$, $j = 1, \ldots, N$, where $\mathcal{E}'$ belongs to the fermion, and a linear term of the polynomial $\Psi_\beta$ ($\Psi_{\dot\beta}$) must be a solution of the free fermion equations, $\Psi^{(0)}_\beta = æ'_\alpha \hat{\mathcal{E}}'$ ($\Psi^{(0)}_{\dot\beta} = \lambda'_{\dot\alpha}\hat{\mathcal{E}}'$). Four-momentum of the fermion, $k'_{\alpha\dot\alpha}$, in both cases decomposes as $k'_{\alpha\dot\alpha} = æ'_\alpha \lambda'_{\dot\alpha}$. The hat above $\mathcal{E}'$ in this case indicates that $\hat{\mathcal{E}}'$ includes a vector from the gauge group representation. Such solutions of Eqs. (30), (31) read as

$$\begin{aligned} \Psi_\beta &= (g^{ptb}|_{(p=æ')})^{-1}\Psi^{(0)}_\beta \\ \Psi_{\dot\beta} &= -i\frac{q^\alpha \partial_{\alpha\dot\beta}}{(q, æ')}(g^{ptb}|_{(p=æ')})^{-1}\hat{\mathcal{E}}' \end{aligned} \tag{32}$$

where $g^{ptb}$ acts on $\hat{\mathcal{E}}'$ in the corresponding representation. Using these expressions one can easily write down the so-called off-shell fermionic current used in ref.[11]. Notice that amplitudes with two on-shell massless fermions and any number of SD gluons vanish.

5. As we mentioned above (cf. ref.([9]), the concept of perturbiner can be generalized to a topologically nontrivial sector. In the latter case it provides a framework for the instanton mediated multi-particle amplitudes. Intuitively, the topologically nontrivial perturbiner is a sort of hybrid of the topologically trivial perturbiner considered above and of the standard instanton solution. Again, it is a polynomial in the same nilpotent variables $\mathcal{E}^j$, but coefficients of the polynomial are now (matrix valued) functions on the Eucleadian space (or their analytical continuations to the Minkowski space, cf. ref.[9]). At $\mathcal{E}^j = 0, j = 1, 2, \ldots$, the topologically nontrivial perturbiner $A_{\dot\alpha}^{iptb}$ is just the instanton,

$$A_{\dot\alpha}^{iptb}|_{(\mathcal{E}^j=0, j=1,\ldots)} = A_{\dot\alpha}^{inst} \tag{33}$$

(which can be considered as a particular case of the restriction property Eq. (10)). All what we need to know about the instanton, $A_{\dot\alpha}^{inst}$, that it can be represented in the twistor-spirit form

$$A_{\dot\alpha}^{inst} = g_{inst}^{-1} \bar\partial_{\dot\alpha} g_{inst} \tag{34}$$

$g_{inst}$ is assumed to be a rational function of the auxiliary variables $p^\alpha$, such that $A_{\dot\alpha}^{inst}$ is a linear homogeneous function of $p^\alpha$. Then the SD topologically nontrivial perturbiner $A_{\dot\alpha}^{iptb}$ is represented in the form

$$A_{\dot\alpha}^{iptb} = (g^{iptb})^{-1} \bar\partial_{\dot\alpha} g^{iptb} \tag{35}$$

and the corresponding $g^{iptb}$ is found to be

$$g^{iptb}(\hat{\mathcal{E}}^1, \hat{\mathcal{E}}^2, \ldots) = g^{inst} g^{ptb}(\hat{\mathcal{E}}_g^1, \hat{\mathcal{E}}_g^2, \ldots) \tag{36}$$

where $\hat{\mathcal{E}}_g^j, j = 1, 2, \ldots$ stand for twisted harmonics

$$\hat{\mathcal{E}}_g^j = (g_{inst}|_{(p=æ^j)})^{-1} \hat{\mathcal{E}}^j g_{inst}|_{(p=æ^j)} \tag{37}$$

and $g^{ptb}$ is defined in Eq. (18).

$g^{iptb}$ from Eq.(36) allows one to reproduce in the instanton sector all the results described above in the topologically trivial case. More detailed account of these issues, as well as calculation of one-loop corrections, will be done elsewhere [14].

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